Editorial review and camera-ready copy by Technical Reports Branch, Harry Diamond Laboratories

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: (14) CM/CCM-78-2
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): (6) Frequency Hopping in a Jamming Environment,
5. TYPE OF REPORT & PERIOD COVERED: (9) Technical Report,
6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): (10) Don J. Torrieri
8. CONTRACT OR GRANT NUMBER(s): (16) DA: 1S665702D671
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Countermeasures/Counter-countermeasures Office, 2800 Powder Mill Road Adelphi, MD 20783
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: Program Ele: 6.57.02.A
11. CONTROLLING OFFICE NAME AND ADDRESS: U.S. Army Materiel Development and Readiness Command Alexandria, VA 22333
12. REPORT DATE: (11) December 1978
13. NUMBER OF PAGES: 50
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office): (12) 51p.
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES: ERADCOM Project: T4980A DRCMS Code: 665702.6710012
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):

Frequency hopping
Jamming
Repeater jamming
Partial-band jamming
Noise jamming
Narrowband jamming
Pseudonoise spread spectrum

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The response of frequency-hopping systems to various types of jamming is examined. Fast and slow hopping are defined in terms of the information bit rate. The impact of coding on the jamming susceptibilities of fast and slow hopping systems is analyzed. The required conditions for effective repeater jamming and → over page

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

410 941

79 04 1 105

LB

effective partial-band jamming are explored. Frequency hopping is compared with pseudonoise spread spectrum. The use of different types of data modulation with frequency hopping is discussed.

FOREWORD

This report gives an analysis of the impact of various types of jamming on frequency-hopping systems. The results are useful in evaluating the performance of many existing and planned systems, including SINCGARS-V. In applying the results, one must be careful to note that the definitions of "fast" and "slow" frequency-hopping systems are natural ones for mathematical analysis, but not necessarily the same as the usage in programs such as SINCGARS-V. Also, one should be aware that engineering and cost constraints in these programs often make system optimization in a mathematical sense undesirable.

The essential feature of "fast" systems, as defined in this report, is that the frequency changes for each transmitted encoded bit of a word. Thus, the analytical results for "fast" systems also apply to a system that is hopping at a low rate, but incorporating bit interleaving over a sufficient number of hopping periods. Of course, error-correcting codes can be used in addition to or without bit interleaving.

If the jammer's available power is less than the product of the number of hopping channels and the signal power, then it is usually advantageous for the jammer to concentrate the power in part of the total bandwidth. However, the degree and the significance of the advantage depend upon whether the performance criterion is the bit error rate or the word error rate. The advantage is also a function of the thermal noise level, word length, coding, and other system parameters. Thus, careful specification of the system and its environment is necessary before drawing conclusions about the relative merit of partial-band jamming.

CONTENTS

FIGURES

FIGURES (Cont'd)

1. INTRODUCTION

Frequency hopping is the periodic changing of the frequency or frequency sets associated with a transmission. Figure 1 illustrates general forms of the transmitter and the receiver. The data modulation can take many forms. Initially, binary frequency-shift keying (FSK) is assumed. Other data modulations are discussed in section 9. Dixon[1] gives a basic introduction to frequency hopping, especially with FSK.

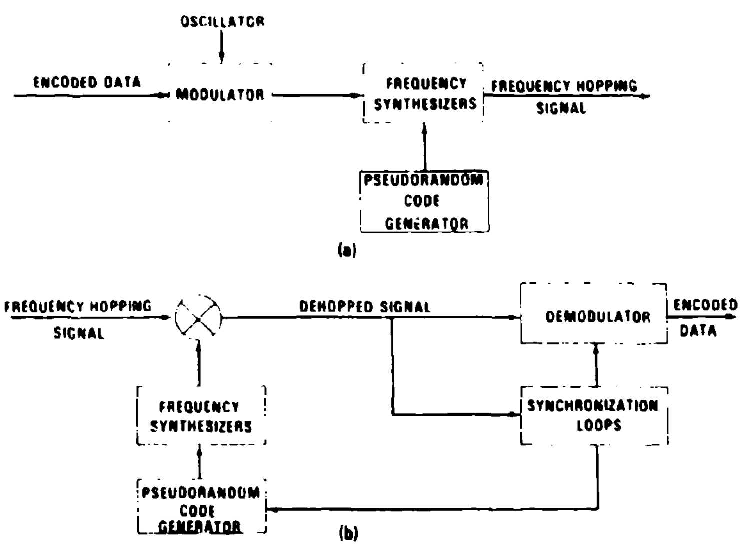

Figure 1. General transmitter (a) and receiver (b) for frequency-hopping system.

When binary FSK is used, each encoded bit (symbol) is transmitted as one of two frequencies, where one frequency represents a logical 1 ("mark"), and the other, a logical 0 ("space"). The pair of possible frequencies is changed periodically by a pseudorandom code. Each change constitutes a hop.

A block diagram for a noncoherent receiving system is shown in figure 2. The two synthesizers produce frequencies which are offset from the two possible received frequencies by constant intermediate frequencies so that only two bandpass filters are needed. After the

[1]*R. C. Dixon, Spread Spectrum Systems, John Wiley and Sons, Inc., New York (1976).*

"dehopping," the demodulation is the same as ordinary noncoherent FSK demodulation. Following a decision with respect to each bit, the final processing involves the decoding of words (groups of code bits) and error detection or correction.

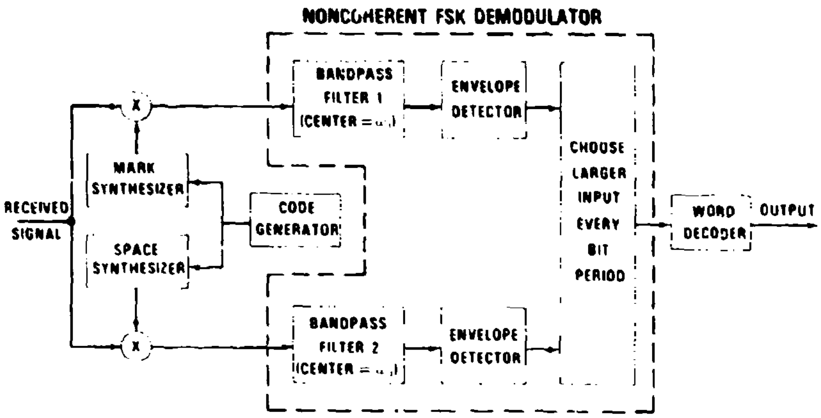

Figure 2. Frequency-hopping receiver.

In this paper, the spectrum occupied by a transmitted bit is called the transmission channel. The spectrum that would be occupied if the logical state represented by the bit were reversed is called the complementary channel. The channels change with the frequency hopping.

The use of two independent synthesizers in figure 2 permits a nonconstant relationship between each pair of frequency channels. As a result, a jammer cannot intercept a frequency, translate it by a constant amount, and jam the complementary channel.

Frequency hopping may be classified as "fast" or "slow." For the mathematical analysis, fast frequency hopping is defined to be hopping at a rate equaling or exceeding the data (message) bit rate. In other words, if the time interval between frequency hops is less than or equal to the time interval of a data bit, then the frequency hopping is said to be fast. Under the opposite conditions, the frequency hopping is said to be slow.

We assume that words of w data bits are to be transmitted at the rate of f_b bits per second. The words are encoded so that C bits are transmitted for each word. To maintain an information rate of f_b, the bits must be transmitted at the rate

$$f_c = \frac{C}{w} f_b \tag{1}$$

bits per second.

Depending upon the code, r or more bits per code word must be in error for a word error to occur at the receiver output. Thus, the probability of a word error is

$$P_w = \sum_{m=r}^{C} P(m), \tag{2}$$

where P(m) is the probability of exactly m bit errors in a word of C bits.

Each frequency-hopping channel requires a bandwidth of approximately $2f_c$ for most of the signal energy to be received. If the total available bandwidth is B_0, then the number of available channels is

$$M = a \frac{B_0}{2f_c} = \frac{aB_0 w}{2f_b C}, \tag{3}$$

where "a" is a parameter which accounts for variation in channel separation and bandwidth other than $2f_c$. In a jamming environment, inadequate channel separation may allow a single narrowband jamming signal to affect two adjacent channels. We assume that the channels associated with the transmission of a word are all distinct. Thus, we require that $2C \leq M$.

The transmission channel, the complementary channel, or both channels associated with a bit may contain jamming energy. We denote the total number of jammed channels by J. By definition, $J \leq M$. For simplicity, we assume that the jamming power is the same in all jammed channels.

In the following analysis, it is always assumed that the synchronization loops of the receiver and the dehopping operate perfectly.

2. FAST FREQUENCY HOPPING

In fast frequency-hopping systems, the frequency changes for each transmitted encoded bit, which is called a chip in this case. Consequently, the jamming environment may change from chip to chip. If slow frequency hopping is used with bit interleaving over a sufficient number of hopping periods, then the jamming environment may change for each encoded bit of a word. Thus, the following analysis is applicable to both fast systems and slow systems with bit interleaving.

Out of the C channels used in transmitting the code word, let k represent the number of transmission channels containing jamming power. Let i represent the number of complementary channels containing jamming power for the same word. Let q represent the number of chips in a word for which both associated channels have jamming power. For these definitions, consistency requires that the following inequalities be satisfied:

$$0 \leq q \leq i \leq C \ ; \quad 0 \leq q \leq k < C \ ; \quad i + k \leq J \ ; \quad i + k - q \leq C \ ;$$

$$C - k \leq M - J \ ; \quad J - k - i \leq M - 2C \quad . \tag{4}$$

Other inequalities required for consistency are implied by those above.

We may decompose P(m) in terms of mutually exclusive and exhaustive events so that

$$P(m) = \sum_{k} \sum_{i} \sum_{q} P(m/k,i,q) P_5 \quad . \tag{5}$$

The summations are carried out over those indices for which the inequalities are satisfied. $P(m/k,i,q)$ is the probability of m chip errors given the occurrence of the event $A(k,i,q)$, which is the event that k transmission channels and i complementary channels are jammed and q chips have both associated channels jammed. P_5 is the probability of event $A(k,i,q)$.

P_5 can be evaluated by combinatorial analysis. We assume that C transmission channels and C complementary channels out of a total of M possible channels have been associated with each word. We assume that the code generator and the frequency synthesizers are designed so that the 2C channels are distinct. The jammer introduces jamming into J randomly chosen channels. Alternatively, we could consider the J jammed channels as fixed and the 2C transmission and complementary channels as randomly chosen for each word. In either case, we can derive the same formula for P_5. However, the former approach yields a simpler derivation.

There are $\binom{M}{J}$ ways to choose the J jammed channels out of the M total channels. The number of ways in which the event $A(k,i,q)$ can occur may be determined by specifying a four-step process. There are $\binom{C}{k}$

ways to choose the k jammed transmission channels of a word. Having chosen these channels, there are $\binom{k}{q}$ ways to choose those channels which have jammed complementary channels associated with them. There are $\binom{C-k}{i-q}$ ways to choose the i-q complementary channels which are jammed but are not associated with jammed transmission channels. The final step in the process is to select the J-k-i jammed channels out of the M-2C channels which are not associated with the word. This selection can be accomplished in $\binom{M-2C}{J-k-i}$ ways. Thus, the probability of event A(k,i,q) is

$$P_5 = \frac{\binom{C}{k}\binom{k}{q}\binom{C-k}{i-q}\binom{M-2C}{J-k-i}}{\binom{M}{J}} \quad . \tag{6}$$

We define $B(\alpha,\beta,\gamma)$ for $\alpha + \beta + \gamma \leq m$ as the event that α errors occur in the chips for which only the transmission channel is jammed, β errors occur in the chips for which only the complementary channel is jammed, and γ errors occur in the chips for which both associated channels are jammed. For consistency and notational convenience, we require that

$$0 \leq \alpha \leq k - q \; ; \quad 0 \leq \beta \leq i - q \; ; \quad 0 \leq \gamma \leq q \; ; \quad \alpha + \beta + \gamma \leq m \; ;$$

$$C - (k - q) - (i - q) - q \geq m - (\alpha + \beta + \gamma) \quad . \tag{7}$$

Other inequalities required for consistency are implied by those above. We make the decomposition

$$P(m/k,i.q) = \sum_{\alpha} \sum_{\beta} \sum_{\gamma} P_4 P(\alpha,\beta,\gamma/k,i,q) \quad . \tag{8}$$

The summations are carried out over those indices for which the inequalities are satisfied. $P(\alpha,\beta,\gamma/k,i,q)$ is the probability of $B(\alpha,\beta,\gamma)$ given A(k,i,q), and P_4 is the probability of m bit errors given the event $A(k,i,q) \cap B(\alpha,\beta,\gamma)$.

Given the latter event, m chips in a word are in error if there are $m - \alpha - \beta - \gamma$ errors among the $C - k - i + q$ chips for which there is no jamming power in either associated channel. Assuming that the chip errors are independent,

$$P_4 = \binom{C-k-i+q}{m-\alpha-\beta-\gamma} S_0^{m-\alpha-\beta-\gamma} \left(1 - S_0\right)^{C-k-i+q-m+\alpha+\beta+\gamma} , \qquad (9)$$

where S_0 is the probability of a chip error when neither associated channel is jammed. It follows from elementary probability that

$$P(\alpha,\beta,\gamma|k,i,q) = P_1 P_2 P_3 , \qquad (10)$$

where

$$P_1 = \binom{k-q}{\alpha} S_t^{\alpha} \left(1 - S_t\right)^{k-q-\alpha} , \qquad (11)$$

$$P_2 = \binom{i-q}{\beta} S_c^{\beta} \left(1 - S_c\right)^{i-q-\beta} , \qquad (12)$$

$$P_3 = \binom{q}{\gamma} S_2^{\gamma} \left(1 - S_2\right)^{q-\gamma} . \qquad (13)$$

In these equations, S_t is the probability of a chip error when only the transmission channel is jammed; S_c is the probability of a chip error when only the complementary channel is jammed; and S_2 is the probability of a chip error when both channels associated with a chip are jammed.

Combining the above definitions, equations, and inequalities, we obtain the probability of a word error,

$$P_w = \sum_{m=r}^{C} \sum_{k=k_0}^{k_1} \sum_{i=i_0}^{i_1} \sum_{q=q_0}^{q_1} \sum_{\alpha=0}^{\alpha_1} \sum_{\beta=0}^{\beta_1} \sum_{\gamma=\gamma_0}^{\gamma_1} P_1 P_2 P_3 P_4 P_5 , \qquad (14)$$

where

$k_0 = \max\ (0,\ C + J - M)$, $\qquad q_1 = \min\ (i,\ k)$,

$k_1 = \min\ (C,\ J)$, $\qquad \alpha_1 = \min\ (m,\ k - q)$,

$i_0 = \max\ (0,\ 2C + J - M - k)$, $\qquad \beta_1 = \min\ (m - \alpha,\ i - q)$,

$i_1 = \min\ (C,\ J - k)$, $\qquad \gamma_0 = \max\ (0,\ m - \alpha - \beta - C + k + i - q)$,

$q_0 = \max\ (0,\ k + i - C)$, $\qquad \gamma_1 = \min\ (m - \alpha - \beta,\ q)$.

The summation limits ensure that all the binomial coefficients, $\binom{a}{b}$, are well defined.

Before specifying the equations for S_0, S_t, S_c, and S_2, we derive the expression for the probability of a word error for slow frequency hopping.

3. SLOW FREQUENCY HOPPING

In slow frequency-hopping systems, a frequency hop occurs once every two or more transmitted bits. To simplify the analysis, we assume that the hops coincide with the boundary between two words; that is, no change to a new pair of possible frequencies can occur during the transmission of a word. Consequently, the jamming environment is the same for each bit in a single word.

Let D_0 denote the event that neither of the two frequencies associated with a word is jammed, D_1 denote the event that one frequency is jammed, and D_2 denote the event that both frequencies are jammed. We can make the decomposition,

$$P(m) = \sum_{n=0}^{2} P\left(m/D_n\right)P\left(D_n\right) \quad , \tag{15}$$

where $P\left(m/D_n\right)$ is the probability of m bit errors in a word, given that event D_n occurs, and $P\left(D_n\right)$ is the probability of event D_n. From elementary combinatorial considerations,

$$P\left(D_n\right) = \frac{\binom{2}{n}\binom{M-2}{J-n}}{\binom{M}{J}} \quad , \; n \leq J, \; J - n \leq M - 2, \; 0 \leq n \leq 2 \; ,$$

$$P\left(D_n\right) = 0, \text{ otherwise} \quad . \tag{16}$$

Assuming independence of bit errors, we have

$$P\left(m/D_n\right) = \binom{C}{m}\left(1 - S_n\right)^{C-m} S_n^m \; , \quad n = 0, 1, 2 \quad , \tag{17}$$

where S_n is the probability of a bit error, given the occurrence of event D_n. Thus, the symbols S_0 and S_2 have the same meaning as they did in the previous section. In terms of the bit error probabilities defined for fast frequency hopping,

$$S_1 = \frac{1}{2}\left(S_t + S_c\right) \quad . \tag{18}$$

This relation follows from the assumption that it is equally likely for either of the two channels to be jammed. Combining equations (2) and (15) through (17), we obtain

$$P_w = \sum_{m=r}^{C} \sum_{n=n_0}^{n_1} \frac{\binom{2}{n}\binom{M-2}{J-n}\binom{C}{m}}{\binom{M}{J}} \left(1 - S_n\right)^{C-m} S_n^m \quad , \tag{19}$$

where

$$n_0 = \max\,(0,\ J + 2 - M), \quad n_1 = \min\,(2,\ J) \quad .$$

The equation for P_w is considerably simpler for slow frequency hopping than for fast frequency hopping.

The probability P_c of a bit error is obtained by setting $C = r = 1$ in equation (19). The result is

$$P_c = \sum_{n=n_0}^{n_1} \frac{\binom{2}{n}\binom{M-2}{J-n}}{\binom{M}{J}} S_n \quad , \tag{20}$$

where

$$n_0 = \max\,(0,\ J + 2 - M), \quad n_1 = \min\,(2,\ J) \quad .$$

Setting $C = r = 1$ in equation (14), we obtain the same expression for P_c after a considerable amount of algebra. Thus, although the word error rates differ, the bit error rates for slow and fast frequency hopping are identical.

We now derive formulas for the conditional bit error rates (S_0, S_t, S_c, and S_2) under various assumptions.

4. CONDITIONAL BIT ERROR RATES

After the received signal has been "dehopped," it is demodulated by an FSK demodulator, as shown in figure 1. Thus, the conditional bit error rates can be evaluated from the theory of FSK demodulation in the presence of jamming. Let N_1 and N_2 represent the power levels of the bandlimited, white Gaussian noise entering the transmission and complementary channels of a receiver, respectively. Some of the noise power, denoted by N_t, is due to thermal and background noise, while the remainder is due to barrage or spot jamming that is modeled as bandlimited, white Gaussian noise. This type of jamming is called noise jamming. Since the jamming is statistically independent of the thermal noise,

$$N_1 = N_t + N_{j1} \quad ,$$

$$N_2 = N_t + N_{j2} \quad . \tag{21}$$

Let R_j represent the average power at the receiver in a narrowband, angle-modulated, jamming signal of the form

$$J(t) = B \cos [\omega t + \phi(t)] \quad , \tag{22}$$

where $\phi(t)$ is a narrowband stochastic process with a uniform distribution at each point in time and ω is the angular frequency. We denote this type of jamming as narrowband jamming. Let R_s represent the average power at the receiver in the intended transmission. Suppose $J(t)$ passes through the transmission channel, but not the complementary channel. It is shown in appendix A that the probability of a bit error is

$$S_t = \frac{N_2}{N_1 + N_2} \exp\left(- \frac{R_s + R_j}{N_1 + N_2}\right) I_0 \left(\frac{2\sqrt{R_s R_j}}{N_1 + N_2}\right) , \tag{23}$$

where $I_0(x)$ is the modified Bessel function of the first kind and zero order. If the complementary channel of a bit is jammed by $J(t)$, but the transmission channel is not, then the probability of a bit error is

$$S_c = Q\left[\left(\frac{2R_j}{N_1 + N_2}\right)^{\frac{1}{2}} , \left(\frac{2R_s}{N_1 + N_2}\right)^{\frac{1}{2}}\right] - \frac{N_1}{N_2} S_t \quad , \tag{24}$$

where the Q-function is defined by

$$Q(\alpha,\beta) = \int_{\beta}^{\infty} x \exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x)\, dx \quad . \tag{25}$$

In the applications, we are primarily interested in two special cases. In one case, we assume that noise jamming, if present, is uniformly distributed over all frequency-hopping channels so that $N_1 = N_2$. In the other case, we assume that narrowband jamming is absent, but that noise jamming is present in some of the available channels; thus, we set $R_j = 0$.

When $N_1 = N_2$, equations (23) and (24) simplify slightly:

$$S_t = \frac{1}{2} \exp\left(-\frac{R_s + R_j}{2N_1}\right) I_0\left(\frac{\sqrt{R_s R_j}}{N_1}\right), \quad N_1 = N_2 \quad , \tag{26}$$

$$S_c = Q\left[\left(\frac{R_j}{N_1}\right)^{1/2}, \left(\frac{R_s}{N_1}\right)^{1/2}\right] - S_t, \quad N_1 = N_2 \quad . \tag{27}$$

Suppose narrowband, angle-modulated jamming signals of equal power enter both receiver bandpass filters and that $N_1 = N_2$. Using equation (A-27) we find that the probability of a bit error is

$$\begin{aligned} S_2 = \frac{1}{2\pi} \int_0^{2\pi} dx \Bigg\{ & Q\left[\left(\frac{R_j}{N_1}\right)^{1/2}, \left(\frac{R_s + R_j + 2\sqrt{R_s R_j}\cos x}{N_1}\right)^{1/2}\right] \\ & - \frac{1}{2} \exp\left[-\frac{R_s + 2R_j + 2\sqrt{R_s R_j}\cos x}{2N_1}\right] \\ & \times I_0\left[\frac{\sqrt{R_j}\left(R_s + R_j + 2\sqrt{R_s R_j}\cos x\right)^{1/2}}{N_1}\right]\Bigg\}, \quad N_1 = N_2 \quad . \end{aligned} \tag{28}$$

If narrowband jamming energy is absent from both the transmission and the complementary channels of a bit, $R_j = 0$. If $N_1 = N_2$, then equation (26) yields

$$S_0 = \frac{1}{2} \exp\left(-\frac{R_s}{2N_1}\right), \; N_1 = N_2 \quad . \tag{29}$$

If noise jamming also is absent, then $N_1 = N_t$.

When $R_j = 0$, equation (23) yields the probability of a bit error,

$$S_n = \frac{N_2}{N_1 + N_2} \exp\left(-\frac{R_s}{N_1 + N_2}\right) \quad , \; R_j = 0 \; , \tag{30}$$

where N_1 includes the noise jamming power in the transmission channel and N_2 includes the noise jamming power in the complementary channel. If the transmission channel is jammed, but the complementary channel is not, then $N_1 = N_t + N_{j1}$ and $N_2 = N_t$. Thus, the probability of a bit error, given that only the transmission channel is jammed, is

$$S_t = \frac{N_t}{2N_t + N_{j1}} \exp\left(-\frac{R_s}{2N_t + N_{j1}}\right), \quad R_j = 0 \quad . \tag{31}$$

Similarly, the probability of a bit error, given that only the complementary channel is jammed, is

$$S_c = \frac{N_t + N_{j2}}{2N_t + N_{j2}} \exp\left(-\frac{R_s}{2N_t + N_{j2}}\right), \quad R_j = 0 \quad . \tag{32}$$

A comparison of the last two equations shows that jamming the complementary channel causes a higher bit error rate than jamming the transmission channel with the same power ($N_{j1} = N_{j2}$). If both channels are jammed with the same power N_j, then $N_1 = N_2 = N_t + N_j$. Thus, the probability of a bit error in this case is

$$S_2 = \frac{1}{2} \exp\left(-\frac{R_s}{2N_t + 2N_j}\right), \quad R_j = 0 \quad . \tag{33}$$

Jamming both channels may be less effective than jamming the complementary channel alone. For example, equations (32) and (33) indicate that $S_2 < S_c$ if $R_s \simeq N_{j} \simeq N_{j2} >> N_t$. Finally, if neither channel is jammed, the probability of a bit error is

$$S_0 = \frac{1}{2} \exp\left(-\frac{R_s}{2N_t}\right), \quad R_j = 0 \; , \; N_j = 0 \quad . \tag{34}$$

5. REPEATER JAMMING

A repeater jammer, also known as a follower or transponder jammmer, is a device that intercepts a transmission, modulates and amplifies the waveform, and retransmits it at the same center frequency. To be effective against a frequency-hopping system, the jamming energy must reach the victim receiver before it hops to a new set of frequency channels. Thus, the greater the hopping rate, the more protected the frequency-hopping system against a repeater jammer.

Figure 3 depicts the geometrical configuration of communicators and a jammer. For the repeater jamming to be effective, we must have

$$\frac{d_2 + d_3 - d_1}{v} + T_p \leq \eta T_h \, , \tag{35}$$

where v is the velocity of an electromagnetic wave, T_p is the processing time required by the repeater, T_h is the "dwell" time or hopping period, and η is a fraction. This equation states that the time delay due to propagation plus the processing time must not exceed a certain fraction of the hopping period if the jamming is to be effective.

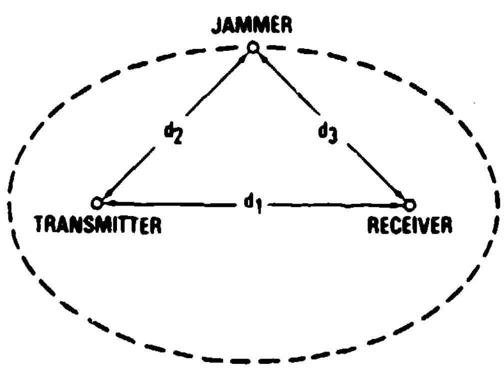

Figure 3. Geometrical configuration of communicators and jammer.

The value of η is determined by the details of the receiver design. Consider fast frequency hopping for which the hopping period is equal to the duration of a coded bit. For the receiver of figure 2, the outputs of the envelope detectors are sampled every bit period. If the sampling occurs in the middle of a bit period, then $\eta = 1/2$. Since it is advantageous to the communicators to force as low a value of the parameter η as possible, the receiver can be designed so that the sampling occurs close to the leading edge of each bit. However, such a strategy increases the degradation due to intersymbol interference and requires greater synchronization accuracy.

Rearranging equation (35), we may write

$$d_2 + d_3 \leq \left(\eta T_h - T_p\right)v + d_1 \quad . \tag{36}$$

If the right-hand side of this inequality is regarded as a constant, then equating the two sides defines an ellipse with the transmitter and the receiver at the two foci. If the repeater jammer is located outside this ellipse, the jamming cannot be effective. Figure 3 shows a jammer located on the boundary of the ellipse.

Assuming that the jammer is located inside the ellipse, we can derive an equation for the word error rate in the victim receiver. Suppose the jammer is able to modulate the intercepted waveform in such a way that the retransmitted waveform is a facsimile of white Gaussian noise over the bandwidth of the victim receiver. If the jamming always enters the transmission channel, which is the only possibility in the fast frequency-hopping case, the probability of a chip error is given by equation (31); that is, in this highly idealized case,

$$P_c = \frac{N_t}{2N_t + N_{j1}} \exp\left(-\frac{R_s}{2N_t + N_{j1}}\right) \quad , \tag{37}$$

where N_{j1} represents the jamming power at the victim receiver.

A more realistic model of what the jammer can accomplish is to assume that the jamming has the form specified by equation (22). If the repeater always jams the transmission channel, the bit error rate is given by equation (26). Thus, in this more realistic case,

$$P_c = \frac{1}{2} \exp\left(-\frac{R_s + R_j}{2N_1}\right) I_0\left(\frac{\sqrt{R_s R_j}}{N_1}\right), \tag{38}$$

where R_j is the jamming power at the victim receiver.

If the bit errors are independent, the probability of a word error is

$$P_w = \sum_{m=r}^{C} \binom{C}{m} \left(1 - P_c\right)^{C-m} P_c^m \quad . \tag{39}$$

From equation (38), we can determine the value of R_j which maximizes P_c for each choice of R_s and N_1. If we assume $\sqrt{R_s R_j} >> N_1$, we may use the asymptotic expression for the Bessel function,

$$I_0(x) \approx \frac{e^x}{\sqrt{2\pi x}} , \quad x >> 1 \quad , \tag{40}$$

in equation (38) to obtain

$$P_c \approx \sqrt{\frac{N_1}{8\pi}} \left(R_s R_j\right)^{-\frac{1}{4}} \exp\left[-\frac{\left(\sqrt{R_s} - \sqrt{R_j}\right)^2}{2N_1}\right] , \quad \sqrt{R_s R_j} >> N_1 \quad . \tag{41}$$

It is now easy to verify with elementary calculus that the optimal value of R_j is $R_j \approx R_s$. Substituting into equation (41), we obtain a simple expression for the bit error rate when the jamming power is optimal,

$$P_c \approx \left(8\pi \frac{R_s}{N_1}\right)^{-\frac{1}{2}} , \quad R_j = R_s >> N_1 \quad . \tag{42}$$

Thus, the repeater jamming is potentially quite effective. However, if the jamming power at the receiver deviates significantly from the optimal value, the effectiveness rapidly decreases, as indicated by equation (41).

When the jamming is modeled as a white Gaussian process, the optimal jamming power is $N_{j1} = R_s - 2N_t$. From equation (37), the corresponding bit error rate is

$$P_c = \left(\frac{R_s}{N_t} e\right)^{-1} , \quad N_{j1} = R_s - 2N_t \quad . \tag{43}$$

The effectiveness of the jamming decreases more slowly than in the preceding case as the jamming power deviates from its optimal value.

As an example, figure 4 shows a plot of equations (37) and (38) when the signal-to-noise ratio is R_s/N_1 = 20 (13 dB) and $N_1 = N_t$. The ordinate is the probability of bit error, P_c, and the abscissa is the jamming-to-signal ratio, R_j/R_s. The figure demonstrates that excessive narrowband jamming power can actually be helpful to the communicators.

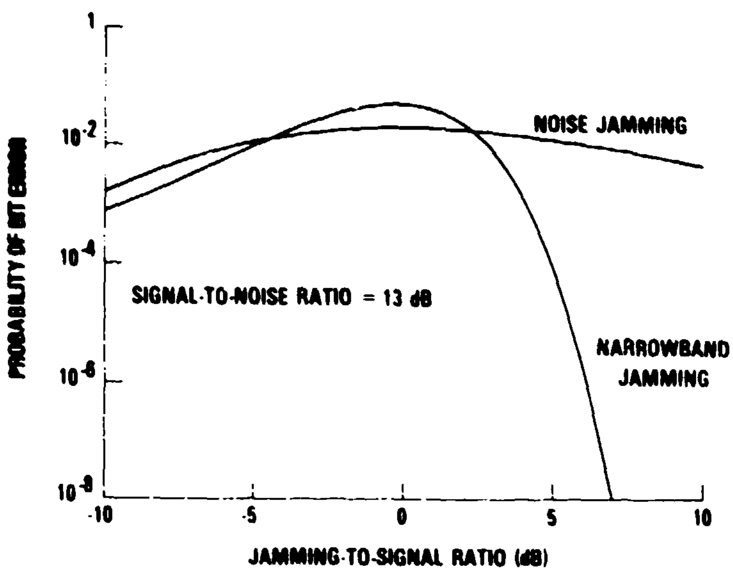

Figure 4. Bit error probability for repeater jamming.

Figures 5 and 6 illustrate how P_c decreases with the signal-to-noise ratio for various fixed values of the jamming-to-noise ratio and the two types of repeater jamming.

In addition to possible geometric and power restrictions, there are other problems and limitations entailed in using a repeater jammer. The repeater must be able to transmit and receive simultaneously at the same frequency. If many communicators are present, the repeater circuitry must be capable of isolating the signal to be repeated. Finally, a repeater jammer usually can disrupt only one frequency-hopping system at a time.

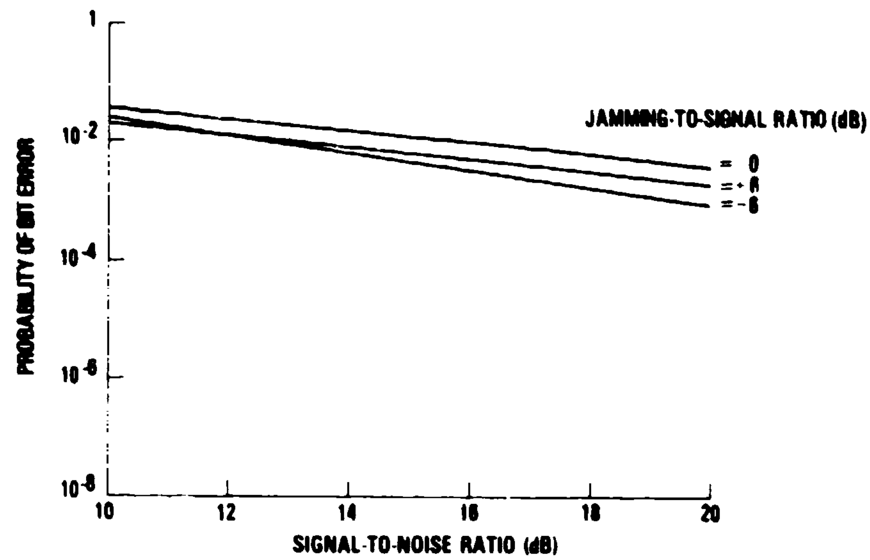

Figure 5. Bit error probability for noise jamming by repeater.

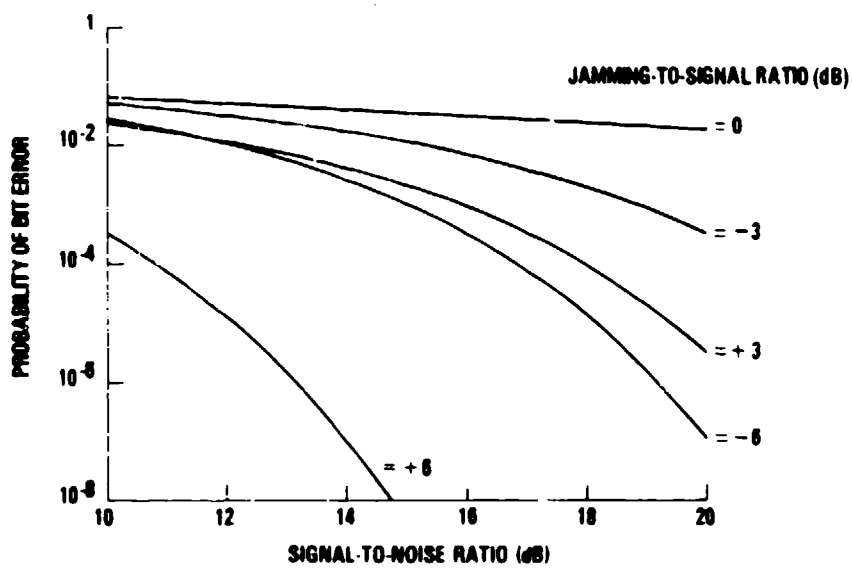

Figure 6. Bit error probability for narrowband jamming by repeater.

6. EFFECT OF CODING

The use of an error-correcting code can dramatically improve the word error rate of a frequency-hopping system in a jamming environment. There are significant differences in the effect of coding on communications by fast frequency hopping and by slow frequency hopping. The reason is that the communicators hop out of a jammed channel after each bit in fast frequency-hopping systems, whereas the communicators dwell in a jammed channel for several bits before hopping in slow frequency-hopping systems. However, if the encoding is followed by bit interleaving in a slow system, then the word error rate is the same as that of the corresponding fast system.

When a block code is used, each uncoded word of w bits is represented by a coded word of C bits. Since C > w, equation (3) indicates that the number of available channels for frequency hopping is reduced; that is,

$$M = \frac{M_u w}{C}, \tag{44a}$$

where M_u is the number of channels which would be available in the absence of coding. Assuming that the time interval of each word is unchanged as a result of the coding, the time interval of a bit is reduced. Consequently, the bandpass filters of figure 2 must have increased bandwidths, and the background noise power entering the filters is given by

$$N_t = \frac{N_{tu} C}{w}, \tag{44b}$$

where N_{tu} is the background noise power which enters in the absence of coding. If the coding is to be effective, its error-correcting capability must be great enough to overcome the degradation implied by equations (44).

As an example, we consider the case in which $M_u = 1000$ and words of length $w = 4$ are to be transmitted in the presence of narrowband jammers with $R_j = R_s$. We use equations (26) through (29) in equations (14) and (19). Figures 7 and 8 show the probability of word error, P_w, as a function of the signal-to-noise ratio per word, wR_s/N_{tu}, for fast and slow frequency hopping, respectively, assuming that the words are uncoded so that C = 4 and r = 1. The curves are plotted for various numbers of jamming signals. Slow hopping produces a slightly lower error rate than fast hopping. Figures 9 and 10 show similar plots when a single-error correcting code is used so that C = 7 and r = 2. The performance of the fast frequency-hopping system improves dramatically as a result of the coding. However, the performance of the slow frequency-hopping system is slightly degraded in the presence of jamming.

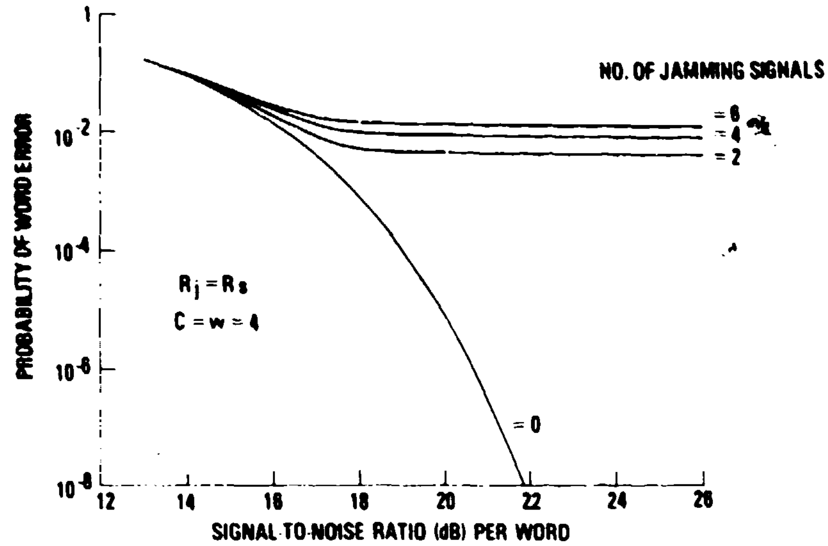

Figure 7. Word error probability for fast hopping and uncoded words.

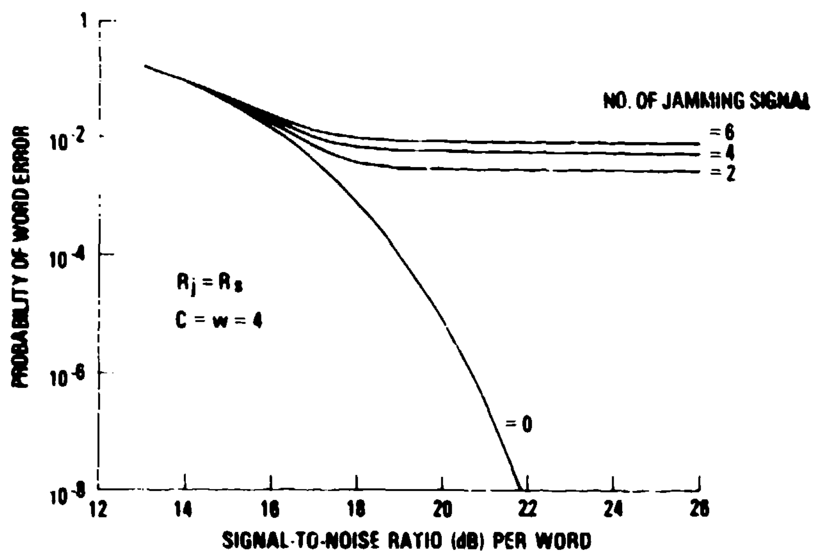

Figure 8. Word error probability for slow hopping and uncoded words.

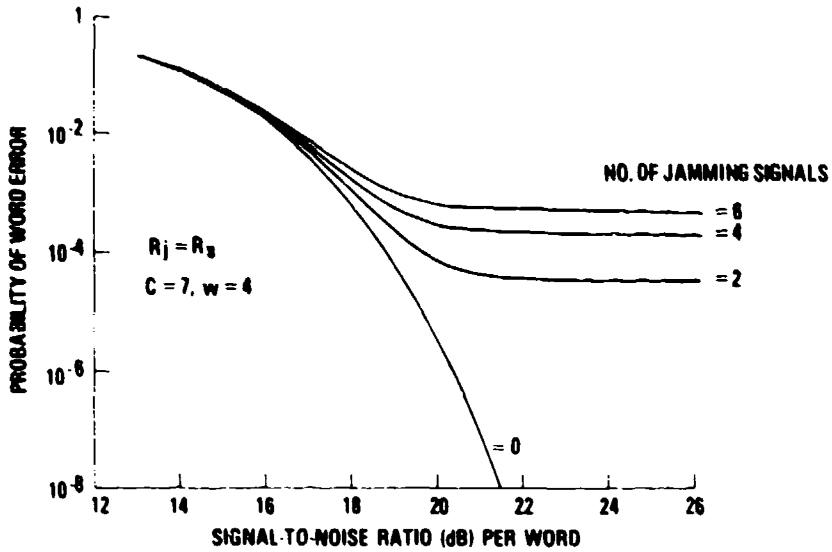

Figure 9. Word error probability for fast hopping and coded words.

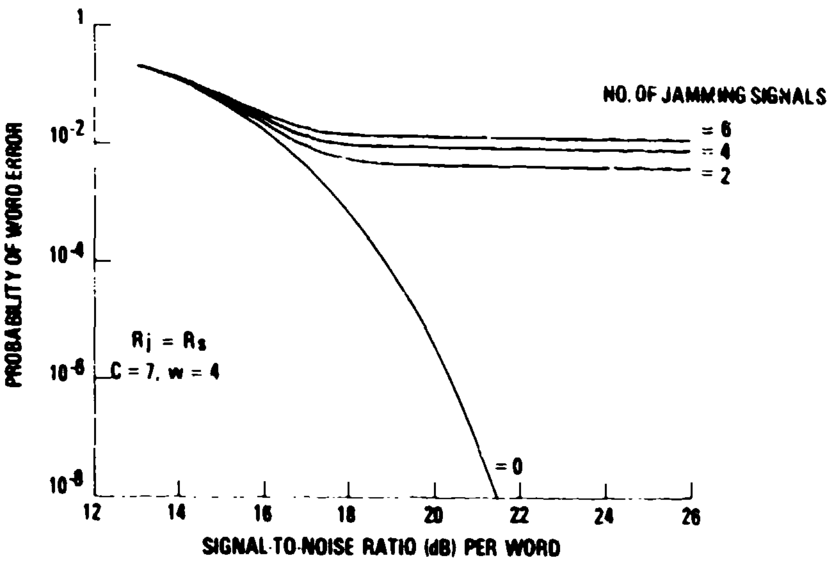

Figure 10. Word error probability for slow hopping and coded words.

The use of repetition coding in a fast system or a slow system with bit interleaving, if feasible, is often very effective in reducing the error rates. Repetition coding consists of transmitting an odd number of code bits for each data bit. The receiver decides the logical state of the data bit according to the logical states of the majority of the received bits. To determine the probability of a data bit error, set $r = (C + 1)/2$ in equation (14) and $w = 1$ in equations (44). As an example, we consider the case in which $M_u = 1000$ and three narrowband jammers with $R_j = R_s$ are present. Figure 11 shows the probability of a data bit error, P_c, as a function of the signal-to-noise ratio per data bit R_s/N_{tu}, for $C = 1, 3, 5$, and 7. Increasing the amount of repetition is helpful only if the received power is sufficiently great.

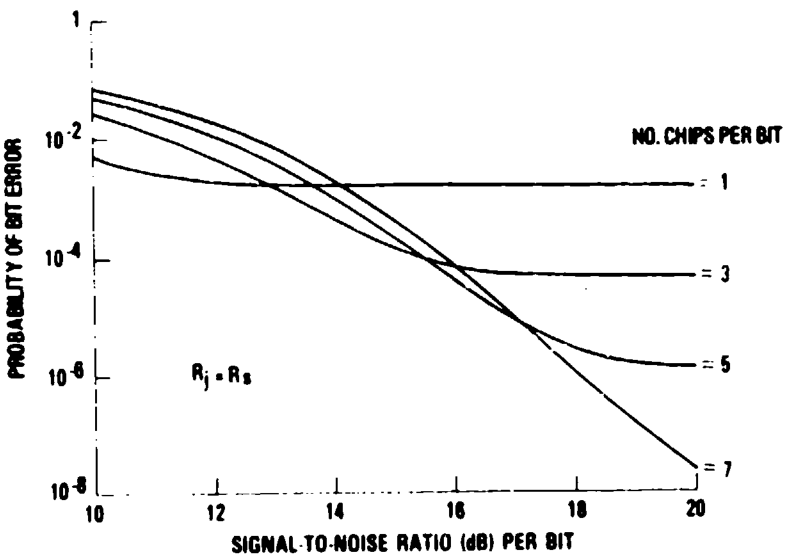

Figure 11. Bit error probability for repetition coding and three narrowband jammers.

7. PARTIAL-BAND JAMMING

If the jammer's available power is less than the product of the number of hopping channels and the signal power, then it is usually advantageous for the jammer to concentrate the power in part of the total bandwidth. As the number of jammed channels is increased, the jamming power available for each of these channels is decreased, assuming that the jammer has a fixed total available power. The jamming power in each jammed channel is

$$N_j = \frac{N_{jt}}{J}\text{, noise jamming,} \tag{45a}$$

$$R_j = \frac{N_{jt}}{J}\text{, narrowband jamming,} \tag{45b}$$

where N_{jt} is the jammer's available power.

To illustrate the implications, we assume that $M_u = 1000$ and $R_s/N_{tu} = 13$ dB and calculate the error probabilities as functions of μ, the fraction of the band that is jammed. We assume that noise jamming is present so that equations (31) through (34) apply. The results are similar for narrowband jamming. For noise jamming, the number of jammed channels is approximated by

$$J = \text{int}(\mu M) = \text{int}\left(\frac{\mu M_u w}{C}\right) ; \tag{46}$$

where int(x) is the largest integer contained in x.

Figure 12 shows the probability of bit error, P_c, versus the fraction of the band that is jammed, μ, for jamming-to-signal ratios of N_{jt}/R_s = 10, 20, and 30 dB. Figures 13 and 14 show the probability of word error, P_w, versus μ for fast and slow frequency hopping, respectively, with $w = 4$, $C = 7$, $r = 2$, and the same power ratios. The optimal band occupancy for the jammer gradually increases as the available power increases. The peaks of the curves in figure 13 are much less pronounced than the peaks in figures 12 and 14. Thus, in this example, if the word error rate is the appropriate measure of system performance, the jammer has little to gain by attempting to implement partial-band jamming against the block-encoded fast frequency-hopping system.

It is intuitively reasonable that the highest bit error rate occurs when the jamming power in the jammed channels is approximately equal to the signal power, since increasing the jamming power beyond this level does not significantly increase the probability of a bit error when the communicators hop into a jammed channel. Consequently, the optimal value of μ is expected to be

$$\mu_0 \approx \frac{N_{jt}}{R_s M} , \quad N_{jt} < R_s M ,$$

$$\mu_0 = 1 , \quad N_{jt} \geq R_s M . \tag{47}$$

It has been verified that these equations are reasonably accurate in many cases.

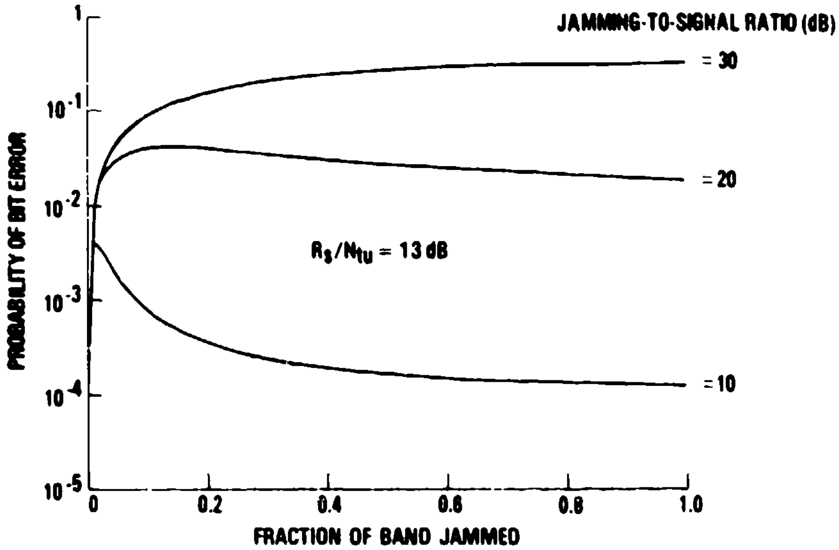

Figure 12. Bit error probability for various ratios of available jamming power to signal power.

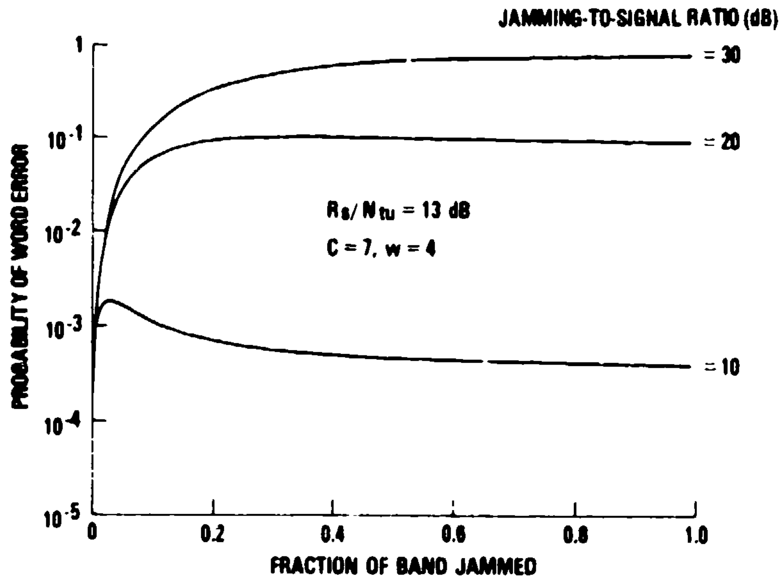

Figure 13. Word error probability for fast hopping and various ratios of available jamming power to signal power.

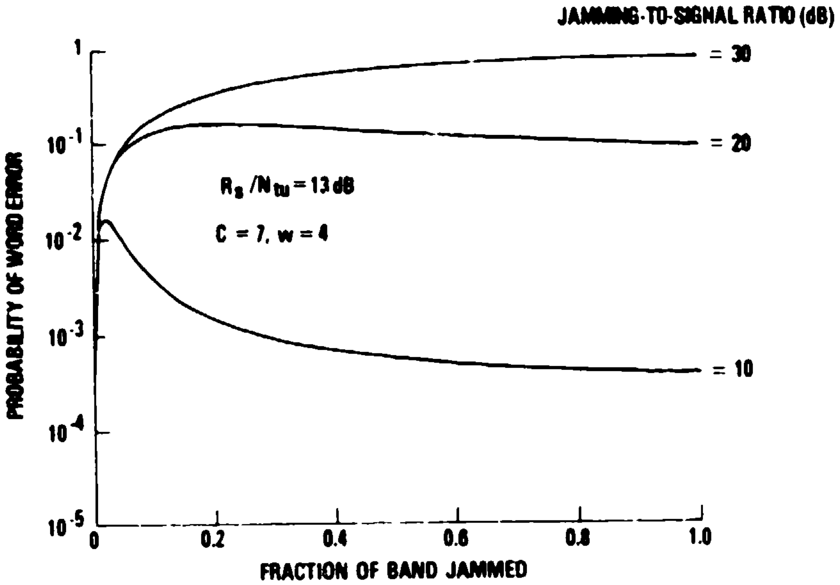

Figure 14. Word error probability for slow hopping and various ratios of available jamming power to signal power.

When some, but not all, of the channels are jammed, the word error rates for block-encoded, slow systems are usually higher than for corresponding fast systems. The errors in the slow systems tend to occur in bursts which may overwhelm the error-correcting capability of the block code. One remedy is to interleave the coded symbols before transmission so that each symbol of a word is associated with a different frequency. After deinterleaving, the error-correcting capability of the block code equals that of the same block code used in a fast system. Thus, by employing additional hardware, slow systems can give the same word error rates as fast systems in the presence of partial-band jamming.

8. COMPARISON WITH PSEUDONOISE SPREAD SPECTRUM

Pseudonoise spread spectrum modulation is often considered an alternative to frequency hopping when communicators must operate in a jamming environment. The probability of a bit error for an ideal coherent pseudonoise system is given by[2]

$$P_c = \frac{1}{2}\,\mathrm{erfc}\left[\left(\frac{B_s}{B_m}\right)\left(\frac{R_s B_m T}{bR_T + N_T}\right)\right]^{\frac{1}{2}} , \qquad (48)$$

[2]*D. J. Torrieri, Communication Warfare, Harry Diamond Laboratories TR-1859 (1978).*

where B_s is the bandwidth over which the transmitted energy is spread, B_m is the bandwidth associated with each message (data) bit, T is the bit duration, R_s is the signal power at the receiver, R_T is the total jamming power in B_s which enters the receiver, N_T is the total background or thermal noise power in B_s, and b is a parameter such that $b \leq 2$. The complementary error function is defined by

$$\operatorname{erfc} x = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-y^2)\, dy \quad . \tag{49}$$

The product $B_m T$ is a constant. The parameter b has a value depending upon the type of jamming. The most destructive jamming occurs when all the jamming energy is concentrated at the center frequency of B_s; in this case, b = 2.

The corresponding probability of a word error, assuming that bit errors occur independently, is

$$P_w = \sum_{m=r}^{C} \binom{C}{m} \left(1 - P_c\right)^{C-m} P_c^m \quad . \tag{50}$$

To compare the pseudonoise and frequency-hopping systems, we assume that the available bandwidth, then transmission power, and the information rate are the same for both systems. Thus, we assume a common value of R_s and

$$N_T = MN_t \ , \ \frac{B_s}{B_m} = M, \quad R_T = JR_j \quad . \tag{51}$$

As a specific example, we further assume the presence of a single narrowband jamming signal at the center frequency of B_s. We allow spectral overlap of the frequency-hopping channels, but we assume that the jamming affects only one channel. Specifically, we assume that

$$J = 1, \quad b = 2, \quad B_m T = 1 \quad . \tag{52}$$

Consequently, equation (48) becomes

$$P_c = \frac{1}{2} \operatorname{erfc} \left(\frac{MR_s}{2R_j + MN_t}\right)^{1/2} \quad . \tag{53}$$

Figures 15 and 16 show comparisons of the probability of word error, P_w, as a function of the signal-to-noise ratio per word, wR_s/N_{tu}, for pseudonoise and fast frequency-hopping systems, with $M_u = 1000$. As the jamming-to-signal ratio, R_j/R_s, is raised beyond a certain point, the frequency-hopping P_w is essentially unchanged, while the pseudonoise P_w degrades rapidly. Block coding is much more helpful for the fast frequency-hopping system than for the pseudonoise system. A fundamental characteristic of frequency-hopping systems is that errors occur primarily when the system hops into a jammed channel. An increase in the jamming energy beyond a certain level has little effect. Pseudonoise systems spread narrowband jamming energy over the total bandwidth. An increase in the jamming energy has a direct effect on the probability of an error. When the total bandwidth of a pseudonoise system is fixed, the potential improvement in performance due to encoding is largely counterbalanced by the decrease in processing gain, which results from the increased transmitted bit rate.

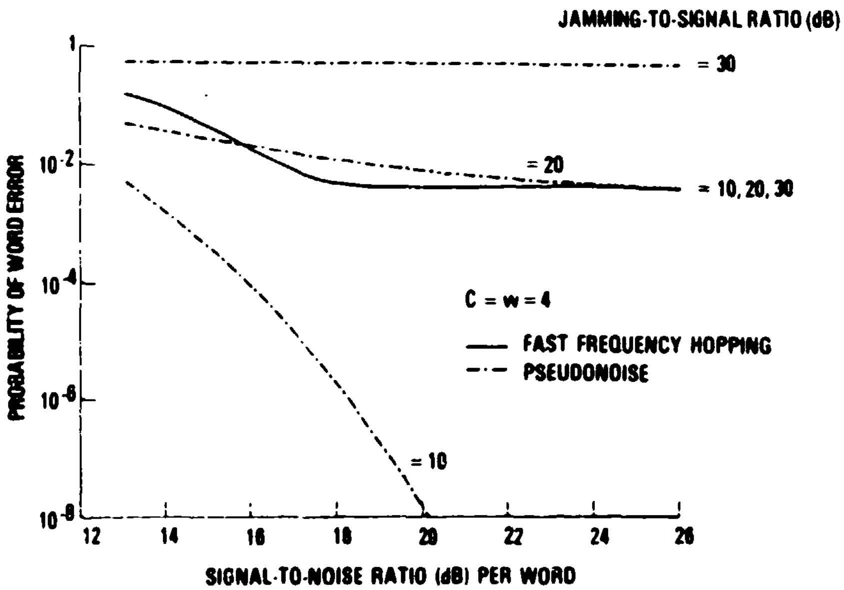

Figure 15. Word error probability for single narrowband jammer and uncoded words.

For successful demodulation in a pseudonoise or frequency-hopping receiver, the code synchronization in the receiver must be accurate to within the time duration of a received bit. For a pseudonoise system, the time duration of a received bit is proportional to $1/B_s$, whereas, for a frequency-hopping system, the time duration of a bit is equal to $1/f_c$. If $B_s >> f_c$, the accuracy requirements for a pseudonoise synchronization system are much more stringent than for a frequency-hopping synchronization system with similar spectral occupancy. Thus, the initial acquisition of frequency-hopping synchronization is generally much more rapid and inherently more jam resistant than the acquisition of pseudonoise synchronization. Of course, pseudonoise systems may employ a frequency-hopping preamble to facilitate acquisition.

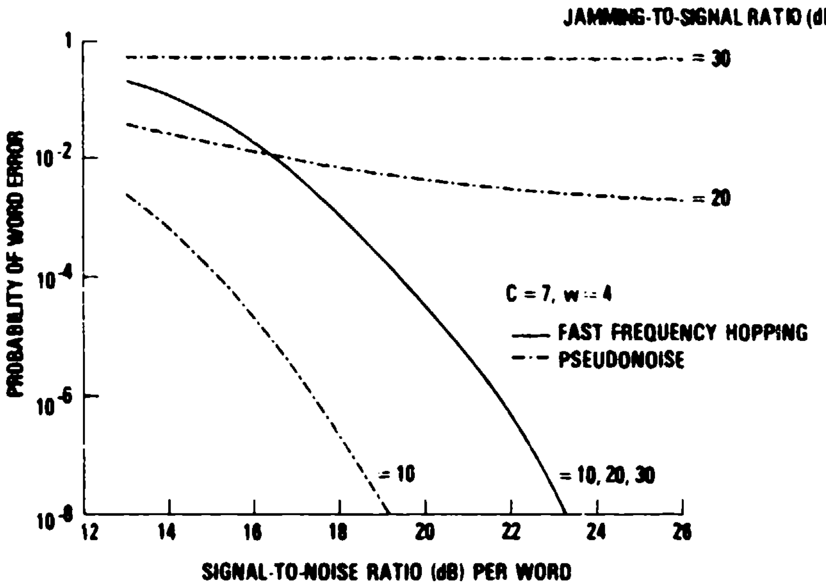

Figure 16. Word error probability for single narrowband jammer and coded words.

Military communication networks frequently require communicators to operate amidst the simultaneous presence of a large number of other network users at widely variable distances and transmitted powers. When there are potentially large power differentials at the receivers between desired and interfering signals, frequency-hopping systems usually perform better than comparable pseudonoise systems. A coordinated frequency-hopping network, in which nearby users transmit on different frequencies and at different times (time hopping), can greatly reduce the mutual interference. Theoretically, interfering signals occurring in channels not being used by the desired signal are noninterfering, regardless of the relative signal strengths. Practically, this capability is limited by the spectral overlap produced by the time-limited transmitted pulses and by spurious spectral components produced by the frequency synthesizers. Appropriate filtering can reduce these deleterious effects.[3]

If it is not feasible to hop at a fast enough rate to eliminate the repeater jamming threat, then a hybrid system combining frequency hopping and pseudonoise spread spectrum may solve the problem.[1] If the autocorrelation function of the pseudorandom code has a sufficiently narrow triangular peak, repeater jamming is ineffective even if the frequency hopping can be easily followed. The hybrid system is attractive also when it is impractical to design a frequency-hopping system with the number of channels needed to use the entire available bandwidth.

9. OTHER DATA MODULATIONS

It has been assumed that the data modulation is impressed on each transmitted pulse by noncoherent FSK. This method has the practical advantages of minimal hardware and synchronization requirements. It is amenable to fast frequency hopping and block coding. However, the intrinsic bit error rates for both binary phase-shift-keying (PSK) and quadriphase-shift-keying (QPSK) systems in the presence of white Gaussian noise are lower than the bit error rate for FSK. In addition, PSK systems require one half and QPSK systems one fourth the bandwidth of FSK systems. Thus, more frequency-hopping channels are available when PSK or QPSK modulation is used instead of FSK. For these reasons, we investigate the effect of using multiple-phase-shift keying (MPSK) as the data modulation.

Because carrier phase coherence must be maintained from hop to hop, it is very difficult to use coherent MPSK for the data modulation in fast frequency-hopping systems and in many slow frequency-hopping systems. A much more practical choice is differentially encoded MPSK, which is demodulated by differentially coherent processing. The differential encoding degrades the intrinsic bit error rate slightly.[4]

---

[1]*R. C. Dixon, Spread Spectrum Systems, John Wiley and Sons, Inc., New York (1976).*

[3]*Spread Spectrum Communications, North Atlantic Treaty Organization Advisory Group for Aerospace Research and Development, National Technical Information Service AD-766-914 (1973).*

[4]*W. C. Lindsey and M. K. Simon, Telecommunication Systems Engineering, Prentice-Hall, Inc., Englewood Cliffs, NJ (1973).*

For MPSK, a convenient set of message signals that may be generated is the polyphase signal set

$$s_i(t) = A \sin\left[\frac{n\pi t}{T_s} + \frac{2\pi(i-1)}{K}\right], \quad 0 \leq t \leq T_s, \quad i = 1, 2, \ldots K, \tag{54}$$

where n is any integer, T_s is the symbol period, and K is the number of different phases or symbols. For $K = 2$, we get PSK; for $K = 4$, we get QPSK. The signal $s_i(t)$ is impressed upon a carrier which hops in frequency.

The symbol error rate for frequency hopping with MPSK can be obtained in a straightforward manner since there is no complementary channel. The probability that the transmission channel is jammed is J/M. Thus, the probability of a symbol error is

$$P_s = \frac{J}{M} S_1 + \left(1 - \frac{J}{M}\right) S_0 \quad , \tag{55}$$

where S_1 is the probability of a symbol error given that the transmission channel is jammed, and S_0 is the probability of a symbol error given that the channel is not jammed. We assume that the only type of jamming present is noise jamming, which occupies some of the available channels. Define

$$R_0 = \frac{R_s}{N_t}\left(B_m T_s\right) \quad ,$$

$$R_1 = \frac{R_s}{N_t + N_j}\left(B_m T_s\right) \quad , \tag{56}$$

where R_s is the signal power, N_t is the sum of the thermal and background noise powers in a channel, B_m is the channel bandwidth, T_s is the symbol duration, and N_j is the jamming power in jammed channels. For differentially encoded PSK, we have

$$S_i = \frac{1}{2} \exp\left(-R_i\right) \quad , \quad i = 0, 1, \quad K = 2 \quad . \tag{57}$$

For differentially encoded MPSK with $K \geq 4$, the exact expression for S_i is complicated. A good approximation is[4]

[4] *W. C. Lindsey and M. K. Simon, Telecommunication Systems Engineering, Prentice-Hall, Inc., Englewood Cliffs, NJ (1973).*

$$S_i = \operatorname{erfc}\left(\frac{R_i}{\sqrt{1 + 2R_i}} \sin \frac{\pi}{K}\right), \quad i = 0, 1, \quad K \geq 4 \quad . \tag{58}$$

Because of its theoretical interest, we consider frequency hopping with coherent PSK in detail. If the time between hops is sufficiently great, it is possible that a phase-locked loop can lock on to the data modulation so that coherent demodulation is accomplished. For slow frequency hopping with block coding, the probability of a word error is determined by reasoning similar to that used in section 3. The result is

$$P_w = \sum_{m=r}^{C} \binom{C}{m}\left[\frac{J}{M} S_1^{\,m} \left(1 - S_1\right)^{C-m} + \left(1 - \frac{J}{M}\right) S_0^{\,m} \left(1 - S_0\right)^{C-m}\right], \tag{59}$$

where the coherent PSK bit error probabilities are

$$S_i = \frac{1}{2} \operatorname{erfc}\left(\sqrt{R_i}\right), \quad i = 0, 1 \quad . \tag{60}$$

Figure 17 depicts the probability of word error, P_w, as a function of the fraction of the band jammed, μ, for various ratios of available jamming-to-signal power and R_s/N_{tu} = 13 dB, C = 7 , w = 4, r = 2, M_u = 1000, and $B_m T_s$ = 1. Equations (44b), (45a), and (46) are used in the calculations. Comparison with figure 14 indicates that slow frequency hopping with coherent PSK is less susceptible to partial-band jamming than slow frequency hopping with noncoherent FSK. If the practical implementation problems can be solved for a hopping rate sufficient to avoid repeater jamming, then coherent PSK is very attractive.

Although it is extremely unlikely that fast frequency hopping with coherent PSK can be effectively implemented, the word error probability is written since it applies to slow frequency hopping with bit interleaving and coherent PSK:

$$P_w = \sum_{m=r}^{C} \sum_{k=k_0}^{k_1} \sum_{\alpha=\alpha_0}^{\alpha_1} \frac{\binom{J}{k}\binom{M-J}{C-k}\binom{C-k}{m-\alpha}\binom{k}{\alpha}}{\binom{M}{C}} \left[S_0^{m-\alpha} S_1^{\alpha} \left(1 - S_0\right)^{C-k-m+\alpha} \times \left(1 - S_1\right)^{k-\alpha}\right], \tag{61}$$

where

$$k_0 = \max(0, C + J - M) \qquad \alpha_0 = \max(0, m - C + k),$$

$$k_1 = \min(C, J), \qquad \alpha_1 = \min(m, k).$$

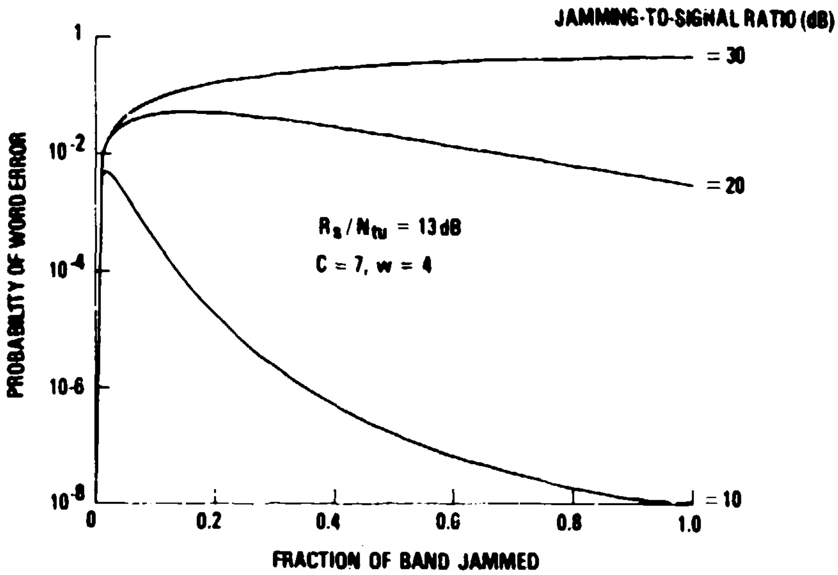

Figure 17. Word error probability for slow hopping, coherent phase-shift-keying data modulation, and various ratios of available jamming power to signal power.

The derivation of this equation is analogous to the derivation of equation (14). Word error rate formulas for differentially encoded PSK are much more involved since errors tend to occur in pairs, and phase reference bits must be included in each word.

Noncoherent FSK detection does not exploit any phase information contained in the received signal. By coherently detecting a continuous-phase-frequency-shift-keying (CPFSK) signal, the phase information can be used to improve the noise performance of the receiver significantly. In fact, the intrinsic bit error rate of CPFSK systems in the presence of white Gaussian noise can be made to approach the bit error rate for coherent PSK systems.[5] A CPFSK signal may be generated by applying the bipolar representation of the input binary data to a voltage-controlled oscillator.

---

[5] *S. Haykin, Communication Systems, John Wiley and Sons, Inc., New York (1978).*

The CPFSK signal occupies much less bandwidth than a conventional FSK signal. Consequently, when CPFSK is used in a frequency-hopping system, the available bandwidth can be partitioned into a larger number of channels, thereby reducing the impact of narrowband jamming signals. Also, the mutual interference in a communication network is decreased.

Unfortunately, there are a number of problems associated with CPFSK. The receiver implementation is much more complex than for noncoherent FSK. A carrier recovery circuit is required for the coherent detection of CPFSK. This circuit, as well as the detector, is affected by the presence of jamming, so that the overall impact of jamming is enhanced. Because of the coherency requirements, it is extremely unlikely that CPFSK can be effectively employed in a fast frequency-hopping system. In a coherent slow system, CPFSK requires an extra phase reference bit to be transmitted every hopping period, causing a degradation from the ideal performance, which may not be negligible unless the hopping rate is much less than the information rate. Finally, the transmission and complementary channels are separated by a fixed amount, making possible the complementary channel jamming by a repeater.

In a white Gaussian noise environment, slow frequency hopping with noncoherent multiple-frequency-shift keying (MFSK) is sometimes advantageous. As the number of different frequencies, K, increases, the symbol error probability for a fixed signal-to-noise ratio per bit decreases as long as the latter exceeds a threshold. However, MFSK with $K > 2$ requires more hardware and occupies more bandwidth per symbol than binary FSK. As the bandwidth occupancy increases, so does the susceptibility to partial-band jamming.

10. CONCLUSIONS

The two major candidates for transmission methods to resist jamming are pseudonoise spread spectrum and frequency hopping. Aside from the issue of cost, frequency hopping is preferable in most military networks for the following reasons.

a. Jamming with sufficiently high power near the center of the spread spectrum will overcome a pseudonoise system. Frequency-hopping systems are inherently insensitive to jamming power increases in a fixed part of the spectrum.

b. If a pseudonoise system does not employ a frequency-hopping preamble for initial synchronization, a jammer has a greatly increased chance to prevent acquisition. After acquisition, pseudonoise systems are more susceptible to loss of synchronization than frequency-hopping systems.

c. Because of multipath propagation, a delayed signal may create serious interference for a pseudonoise system. The interference is negligible for a frequency-hopping system if the demodulator is hopped to a new frequency before any delayed signal can arrive.

d. Because of their insensitivity to high power levels of interference, frequency-hopping systems are usually less degraded by a multiuser environment than pseudonoise systems.

Once it has been decided to use frequency hopping, the next issue is the hopping rate. The repeater jammer constitutes a serious potential threat. This threat can be countered by increasing the hopping rate beyond a certain minimum rate which is a function of the repeater processing time and geometrical considerations. If it is not feasible to design a system with a hopping rate that exceeds this minimum, then a hybrid system combining frequency hopping and pseudonoise spread spectrum is a potential solution to the threat.

Fast frequency hopping is defined as hopping at a rate that exceeds the information bit rate; slow frequency hopping is defined as hopping more slowly than the information bit rate. If the minimum hopping rate for safety from repeater jammers exceeds the information rate, then fast hopping is advisable; otherwise, the choice depends upon other factors.

Due to the difficulty in maintaining carrier phase coherence from hop to hop, the most practical data modulation method for fast frequency-hopping systems appears to be noncoherent binary FSK. Fast systems with binary data modulation are advantageous if the performance measure is the word error rate and if error-correction codes are used. Repetition coding, when used in fast systems or in slow systems with bit interleaving, can significantly lower the bit error rate of these systems with respect to slow ones without interleaving. If block coding is used, the word error rate for fast systems and slow interleaved systems in the presence of partial-band jamming can be considerably lower than for slow systems using block coding alone. Another advantage of fast hopping is that it is more difficult than slow hopping to intercept and process for direction finding.

High hopping rates have several disadvantages in comparison with low rates.

a. The time required for initial synchronization increases with the hopping rate.

b. The frequency synthesizers become a major expense at high hopping rates.

c. The settling time entailed in frequency switching can become a significant portion of the hopping period, causing performance degradation.

d. The mutual interference problem in a communication network and the difficulty of implementing some form of coordination increase with the hopping rate.

In slow frequency-hopping systems, the most practical choice of data modulation appears to be differentially encoded PSK or QPSK, CPFSK, noncoherent MFSK, or noncoherent FSK. Differentially encoded PSK or QPSK and CPFSK minimize the mutual interference problems of communication networks. Noncoherent FSK presents the least practical implementation problem. If feasible, data modulation by coherent PSK is theoretically optimal in terms of bit error rate.

Coding, which can be combined with binary data modulation, is often not sufficient by itself to significantly lower the word error rate of a slow system in the presence of partial-band jamming. However, if bit interleaving also is used, the word error rates of slow systems can equal those of comparable fast systems. Bit interleaving tends to spread clustered bit errors so that error-correction procedures are more effective. The drawback of bit interleaving is the expense, which at least partially offsets the extra expense of frequency synthesizers in competing fast systems.

LITERATURE CITED

(1) R. C. Dixon, Spread Spectrum Systems, John Wiley and Sons, Inc., New York (1976).

(2) D. J. Torrieri, Communication Warfare, Harry Diamond Laboratories TR-1859 (1978).

(3) Spread Spectrum Communications, North Atlantic Treaty Organization Advisory Group for Aerospace Research and Development, National Technical Information Service AD-766-914 (1973).

(4) W. C. Lindsey and M. K. Simon, Telecommunication Systems Engineering, Prentice-Hall, Inc., Englewood Cliffs, NJ (1973).

(5) S. Haykin, Communication Systems, John Wiley and Sons, Inc., New York (1978).

APPENDIX A.--DERIVATIONS OF CONDITIONAL BIT ERROR PROBABILITIES

In this appendix, we determine the bit error rates under various conditions. The noncoherent frequency-shift-keying (FSK) demodulator of figure 1 in the body of the report is separately displayed in figure A-1 for convenience.

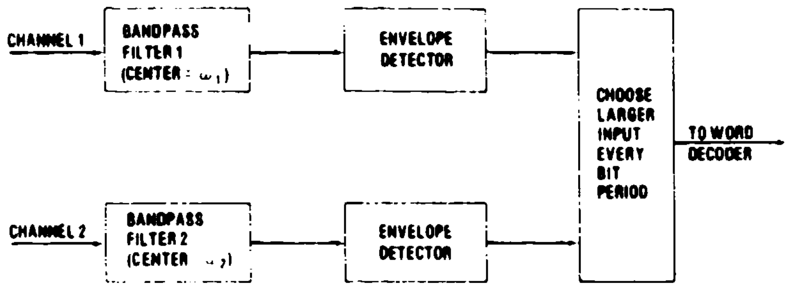

Figure A-1. Noncoherent frequency-shift-keying demodulator.

We assume operation in a white Gaussian noise environment. Thus, at the output of the bandpass filters, the bandlimited white Gaussian noise has the narrowband representation

$$n_i(t) = n_{ci}(t) \cos \omega_i t - n_{si}(t) \sin \omega_i t, \quad i = 1, 2, \tag{A-1}$$

where $n_{ci}(t)$ and $n_{si}(t)$ are independent Gaussian processes with noise powers equal to N_i. Thus, the noise powers in the two branches of the receiver may be different. We assume that jamming signals of the form

$$J_i(t) = B_i(t) \cos \left[\omega_i t + \phi_i(t)\right], \quad i = 1, 2, \tag{A-2}$$

emerge from the bandpass filters centered at ω_i. Suppose a logical 1 signal represented by

$$s_1(t) = A \cos \omega_1 t \tag{A-3}$$

is received and passes through bandpass filter 1 with negligible distortion.

APPENDIX A

The total signals at the outputs of the two bandpass filters are

$$X_1(t) = A_1 \cos \omega_1 t + B_1 \cos\left(\omega_1 t + \phi_1\right) + n_{c1} \cos \omega_1 t - n_{s1} \sin \omega_1 t \quad ,$$

$$X_2(t) = B_2 \cos\left(\omega_2 t + \phi_2\right) + n_{c2} \cos \omega_2 t - n_{c2} \sin \omega_2 t \quad . \tag{A-4}$$

We consider a typical bit interval, which is defined by $0 \le t \le T$. The sampling time, the time at which a bit decision is made, could theoretically be any time within this interval because of the idealized forms assumed in equation (A-3). In practice, a sampling time at the midpoint of the bit interval is likely to provide the best results. We denote the sampling time by T_1. By using trigonometry, equations (A-4) can be written in the form $X_i(t) = R_i(t) \cos\left(\omega_i t + \psi_i\right)$, $i = 1, 2$. Thus, the outputs of the envelope detectors of the two receiver branches at time $t = T_1$ are found to be

$$R_1 = \left(Z_1^2 + Z_2^2\right)^{1/2} \quad ,$$

$$R_2 = \left(Z_3^2 + Z_4^2\right)^{1/2} \quad , \tag{A-5}$$

where the following definitions are made for notational convenience:

$$\begin{aligned} Z_1 &= A + B_1\left(T_1\right) \cos \left[\phi_1\left(T_1\right)\right] + n_{c1}\left(T_1\right) \quad , \\ Z_2 &= B_1\left(T_1\right) \sin \left[\phi_1\left(T_1\right)\right] + n_{s1}\left(T_1\right) \quad , \\ Z_3 &= B_2\left(T_1\right) \cos \left[\phi_2\left(T_1\right)\right] + n_{c2}\left(T_1\right) \quad , \\ Z_4 &= B_2\left(T_1\right) \sin \left[\phi_2\left(T_1\right)\right] + n_{s2}\left(T_1\right) \quad . \end{aligned} \tag{A-6}$$

Since $n(t)$ is assumed to be a zero-mean process, all the noise variables in equation (A-6) are zero mean. Denoting the expected value of Z_i by M_i,

$$\begin{aligned} M_1 &= A + B_1(T_1) \quad \left[\phi_1 \left(T_1\right)\right] \quad , \\ M_2 &= B_1\left(T_1\right) \sin \left[\quad T_1\right)\right] , \\ M_3 &= B_2\left(T_1\right) \cos \left[\phi_2\left(T_1\right)\right] \quad , \\ M_4 &= B_2\left(T_1\right) \sin \left[\phi_2\left(T_1\right)\right] \quad . \end{aligned} \tag{A-7}$$

Assuming that $B_1(T_1)$ and $\phi_1(T_1)$ are given, the joint probability density function (pdf) of Z_1 and Z_2 is

$$g_1(z_1, z_2) = \frac{1}{2\pi N_1} \exp\left[-\frac{(z_1 - M_1)^2 + (z_2 - M_2)^2}{2N_1}\right]. \tag{A-8}$$

If we define $Z_1 = R_1 \cos\theta$ and $Z_2 = R_1 \sin\theta$, it follows that the joint pdf of R_1 and θ is

$$g_2(r_1, \theta_1) = \frac{r_1}{2\pi N_1}$$

$$\times \exp\left(-\frac{r_1^2 - 2r_1M_1 \cos\theta_1 - 2r_1M_2 \sin\theta_1 + M_1^2 + M_2^2}{2N_1}\right), \tag{A-9}$$

$$r_1 \geq 0, \quad |\theta_1| < \pi.$$

The pdf of the envelope R_1 is obtained by integration over θ_1. First we note that the modified Bessel function of the first kind and zero order satisfies

$$I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} \exp\left[x \cos(u + v)\right] du \tag{A-10}$$

regardless of the value of v. Consequently, after suitable trigonometric manipulation, the integral of equation (A-9) over θ_1 can be reduced to

$$f_1(r_1) = \frac{r_1}{N_1} \exp\left(-\frac{D_1^2 + r_1^2}{2N_1}\right) I_0\left(\frac{D_1 r_1}{N_1}\right); \quad r_1 \geq 0, \tag{A-11}$$

where we define

$$D_1^2 = M_1^2 + M_2^2 = A^2 + B_1^2(T_1) + 2AB_1(T_1) \cos\left[\phi_1(T_1)\right]. \tag{A-12}$$

APPENDIX A

In a similar manner, the output at time $t = T_1$ of the envelope detector in the lower branch of the FSK demodulator has the pdf given by

$$f_2\left(r_2\right) = \frac{r_2}{N_2} \exp\left(-\frac{B_2^2 + r_2^2}{2N_2}\right) I_0\left(\frac{B_2 r_2}{N_2}\right), \quad r_2 \geq 0 \quad . \tag{A-13}$$

Since $s_1(t)$ has been transmitted, an error occurs if $R_2 > R_1$. Thus, the probability of an error is

$$P(E/1) = \int_0^\infty f_1\left(r_1\right) \left[\int_{r_1}^\infty f_2\left(r_2\right) dr_2\right] dr_1 \quad . \tag{A-14}$$

Substituting equations (A-11) and (A-13) into equation (A-14), we obtain

$$P(E/1) = \int_0^\infty q\left(\frac{D_1}{\sqrt{N_1}}, x\right) Q\left(\frac{B_2}{\sqrt{N_2}}, \frac{\sqrt{N_1}\, x}{\sqrt{N_2}}\right) dx \quad , \tag{A-15}$$

where we have defined the Rician function

$$q(\alpha, x) = x \exp\left(-\frac{x^2 + \alpha^2}{2}\right) I_0(\alpha x) \tag{A-16}$$

and the Q-function

$$Q(\alpha,\beta) = \int_\beta^\infty q(\alpha, x)\, dx \ . \tag{A-17}$$

The integral in equation (A-15) can be evaluated by using the following identity given by Helstrom:[1]

$$\int_0^\infty q(a, x)\, Q(b, rx)\, dx = Q\left(v_2, v_1\right) - \frac{r^2}{1 + r^2} \times \exp\left[-\frac{a^2 r^2 + b^2}{2\left(1 + r^2\right)}\right] I_0\left(\frac{abr}{1 + r^2}\right) \quad , \tag{A-18}$$

[1] *C. Helstrom, Statistical Theory of Signal Detection, 2nd ed., Pergamon Press, Elmsford, NY (1968).*

where

$$v_1 = ar(1 + r^2)^{-\frac{1}{2}}, \quad v_2 = b(1 + r^2)^{-\frac{1}{2}} \quad .$$

Carrying out the algebra, we obtain

$$P(E/1) = Q\left(\frac{B_2}{\sqrt{N_1 + N_2}}, \frac{D_1}{\sqrt{N_1 + N_2}}\right) - \frac{N_1}{N_1 + N_2} \exp\left[-\frac{B_2^2 + D_1^2}{2\left(N_1 + N_2\right)}\right]$$

$$\times I_0\left(\frac{B_2 D_1}{N_1 + N_2}\right) \quad . \tag{A-19}$$

This expression gives $P(E/1)$ for fixed values of the $B_i(T_1)$ and $\phi_1\left(T_1\right)$. From bit interval to bit interval, these parameters generally vary in value. If these parameters are modeled as random variables, an aggregate $P(E/1)$ can be calculated by integrating the product of equation (A-19) and the joint pdf of the $B_i\left(T_1\right)$ and $\phi_1\left(T_1\right)$. To obtain reasonably simple results, we assume that narrowband angle-modulated jamming is present. Thus, we assume that $B_i(t) = B_i\left(T_1\right) = B_i$, a constant. If $\phi_1(t)$ is nonsynchronous with the carrier frequency of $s_1(t)$, it is logical to model $\phi_1\left(T_1\right)$ as uniformly distributed from 0 to 2π radians. Thus, the aggregate probability of error, given that $s_1(t)$ was transmitted, is

$$\overline{P}(E/1) = \frac{1}{2\pi} \int_0^{2\pi} P(E/1)\, d\phi_1 \quad . \tag{A-20}$$

When a logical 0 represented by

$$s_2(t) = A \cos \omega_2 t \tag{A-21}$$

is received, the bit error probabilities can be determined by an analogous procedure. Defining

$$D_2^2 = A^2 + B_2^2\left(T_1\right) + 2AB_2\left(T_1\right) \cos\left[\phi_2\left(T_1\right)\right] \quad , \tag{A-22}$$

we obtain

$$P(E/2) = Q\left(\frac{B_1}{\sqrt{N_1 + N_2}}, \frac{D_2}{\sqrt{N_1 + N_2}}\right) - \frac{N_2}{N_1 + N_2} \exp\left[-\frac{B_1^2 + D_2^2}{2\left(N_1 + N_2\right)}\right]$$

$$\times \; I_0\left(\frac{B_1 D_2}{N_1 + N_2}\right) \quad . \tag{A-23}$$

If $B_i(t) = B_i\left(T_1\right) = B_i$, a constant, and $\phi_2\left(T_1\right)$ is uniformly distributed, then

$$\overline{P}(E/2) = \frac{1}{2\pi} \int_0^{2\pi} P(E/2)\, d\phi_2 \; . \tag{A-24}$$

If the transmission of a logical 1 or 0 is equally likely, the probability of a bit error is

$$\overline{P}(E) = \frac{1}{2}\,\overline{P}(E/1) + \frac{1}{2}\,\overline{P}(E/2) \quad . \tag{A-25}$$

Substitution of the previous equations into equation (A-25) yields

$$\overline{P}(E) = \frac{1}{4\pi} \int_0^{2\pi} dx \left\{ Q\left[\frac{B_2}{\sqrt{N_1 + N_2}}, \frac{D_1(x)}{\sqrt{N_1 + N_2}}\right] + Q\left[\frac{B_1}{\sqrt{N_1 + N_2}}, \frac{D_2(x)}{\sqrt{N_1 + N_2}}\right] \right.$$

$$- \frac{N_1}{N_1 + N_2} \exp\left[-\frac{B_2^2 + D_1^2(x)}{2\left(N_1 + N_2\right)}\right] I_0\left[\frac{B_2 D_1(x)}{N_1 + N_2}\right] - \frac{N_2}{N_1 + N_2} \tag{A-26}$$

$$\left. \times \exp\left[-\frac{B_1^2 + D_2^2(x)}{2\left(N_1 + N_2\right)}\right] I_0\left[\frac{B_1 D_2(x)}{N_1 + N_2}\right] \right\},$$

where

$$D_i^2(x) = A^2 + B_i^2 + 2AB_i \cos x \; , \quad i = 1, 2 \quad .$$

This equation is a slight generalization of one derived by Pettit.[2]

---

[2]*R. Pettit, Error Probability for NCFSK with Linear FM Jamming, IEEE Trans. Aerosp. Electron. Syst.,* AES-8 *(September 1972), 609-614.*

The various conditional bit error probabilities can be calculated from the above equations. The probability of a bit error given that an equal amount of narrowband jamming power enters both channels, S_2, is determined by setting $B_1 = B_2 = \sqrt{2R_j}$ in equation (A-26), where R_j is the jamming power. Setting $A = \sqrt{2R_s}$, where R_s is the received signal power, we obtain

$$S_2 = \frac{1}{2\pi}\int_0^{2\pi} dx \left\{ Q\left[\left(\frac{2R_j}{N_1 + N_2}\right)^{\frac{1}{2}}, \frac{D(x)}{\sqrt{N_1 + N_2}}\right] - \frac{1}{2}\exp\left[-\frac{2R_j + D^2(x)}{2\left(N_1 + N_2\right)}\right] I_0\left[\frac{\sqrt{2R_j}\,D(x)}{N_1 + N_2}\right]\right\} , \qquad \text{(A-27)}$$

where

$$D^2(x) = 2R_s + 2R_j + 4\sqrt{R_s R_j}\cos x \ .$$

The probability of a bit error given that narrowband jamming enters the transmission channel only, S_t, is determined by setting $B_2 = 0$, $B_1 = \sqrt{2R_j}$, and $S_t = \bar{P}(E/1)$. We use the facts that $I_0(0) = 1$ and

$$Q(0, \beta) = \exp\left(-\frac{\beta^2}{2}\right) \qquad \text{(A-28)}$$

to obtain, from equations (A-19) and (A-20),

$$S_t = \frac{N_2}{2\pi\left(N_1 + N_2\right)}\int_0^{2\pi} \exp\left(-\frac{R_s + R_j + 2\sqrt{R_s R_j}\cos x}{N_1 + N_2}\right) dx \ . \qquad \text{(A-29)}$$

Using equation (A-10) yields

$$S_t = \frac{N_2}{N_1 + N_2}\exp\left[-\frac{R_s + R_j}{N_1 + N_2}\right] I_0\left(\frac{2\sqrt{R_s R_j}}{N_1 + N_2}\right) . \qquad \text{(A-30)}$$

In this equation, N_1 is associated with the transmission channel, and N_2 is associated with the complementary channel.

APPENDIX A

The probability of a bit error given that narrowband jamming enters the complementary channel only, S_c, is determined by setting $B_1 = 0$, $B_2 = \sqrt{2R_j}$, and $S_c = \bar{P}(E/1)$. Since $D_1 = A = \sqrt{2R_s}$, $\bar{P}(E/1) = P(E/1)$. Thus, retaining our interpretation of N_1 and N_2, we get

$$S_c = Q\left[\left(\frac{2R_j}{N_1 + N_2}\right)^{½}, \left(\frac{2R_s}{N_1 + N_2}\right)^{½}\right] - \frac{N_1}{N_1 + N_2} \exp\left[-\frac{R_s + R_j}{N_1 + N_2}\right]$$

$$\times I_0\left(\frac{2\sqrt{R_s R_j}}{N_1 + N_2}\right) . \qquad \text{(A-31)}$$

The probability of a bit error given that narrowband jamming is absent, S_0, is determined by setting $R_j = 0$ in equation (A-27) and using equation (A-28). The result is

$$S_0 = \frac{1}{2} \exp\left(-\frac{R_s}{N_1 + N_2}\right) . \qquad \text{(A-32)}$$

This completes our derivations of the conditional bit error probabilities.

DISTRIBUTION

DEFENSE DOCUMENTATION CENTER
CAMERON STATION, BUILDING 5
ATTN DDC-TCA (12 COPIES)
ALEXANDRIA, VA 22314

COMMANDER
USA RSCH & STD GP (EUR)
BOX 65
ATTN LTC JAMES M. KENNEDY, JR.
CHIEF, PHYSICS & MATH BRANCH
FPO NEW YORK 09510

COMMANDER
US ARMY MATERIEL DEVELOPMENT
& READINESS COMMAND
ATTN DRXAM-TL, HQ TECH LIBRARY
ATTN DRCDE-DC/FIO, COL J. MIKULA
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

COMMANDER
USA MISSILE & MUNITIONS CENTER
& SCHOOL
ATTN ATSK-CTD-F
REDSTONE ARSENAL, AL 35809

DIRECTOR
US ARMY BALLISTIC RESEARCH LABORATORY
ATTN DRDAR-TSB-S (STINFO)
ABERDEEN PROVING GROUND, MD 21005

COMMANDER
US ARMY RESEARCH OFFICE
ATTN COL A. SIMKUS
P.O. BOX 12211
RESEARCH TRIANGLE PARK, NC 27709

COMMANDER
US ARMY AVIATION RESEARCH &
DEVELOPMENT COMMAND
ATTN MR. R. LWEIS, ACTING
TECHNICAL DIR
ST. LOUIS, MO 63166

COMMANDER
US ARMY TANK-AUTOMOTIVE RESEARCH
& DEVELOPMENT COMMAND
ATTN COL W. PALMER
TARAD LABS
WARREN, MI 48090

COMMANDER
US ARMY MISSILE RESEARCH
& DEVELOPMENT COMMAND
ATTN DR. J. KOBLER,
DIRECTOR TECHNOLOGY LAB
REDSTONE ARSENAL, AL 35809

COMMANDER/DIRECTOR
COMBAT SURVEILLANCE
& TARGET ACQUISITION LABORATORY
ATTN DELCS, COL W. EVANS
FT. MONMOUTH, NJ 07703

DIRECTOR
ELECTRONIC WARFARE LABORATORY
ATTN DELEW, MR. C. HARDIN
FT. MONMOUTH, NJ 07703

DIRECTOR
SIGNALS WARFARE LABORATORY
ATTN DELSW, MR. H. HOVEY
VINT HILL FARMS STATION
WARRENTON, VA 22186

US ARMY COMMUNICATIONS RESEARCH
& DEVELOPMENT COMMAND
ATTN MG H. DICKINSON
FT. MONMOUTH, NJ 07703

COMMANDER
US ARMY INTELLIGENCE
& THREAT ANALYSIS DETACHMENT
ATTN ELECTRONIC WARFARE,
LTC J. V. COLE
ARLINGTON HALL STATION
ARLINGTON, VA 22212

COMMANDER
US ARMY FOREIGN SCIENCE
& TECHNOLOGY CENTER
ATTN DRXST-CE1, MR. EDWARD WEBSTER
ATTN DRXST-CE3, MAJ KENNETH KOPECKY
ATTN DRXST-CE2, MR. WILLIAM YOST
ATTN DRXST-CE, MR. A. RICCIARDELLI
220 SEVENTH STREET NE
CHARLOTTESVILLE, VA 22901

COMMANDER
MISSILE INTELLIGENCE AGENCY
ATTN DRDMI-Y, COL JACK WILSON
REDSTONE ARSENAL
HUNTSVILLE, AL 35809

US ARMY ELECTRONICS RESEARCH
& DEVELOPMENT COMMAND
ATTN WISEMAN, ROBERT S., DR., DRDEL-CT
ATTN PAO
ATTN HOYT, L., COL, DRDEL-DF
ATTN MURRAY, G., COL, DRDEL-AP-CCM
(6 COPIES)

DISTRIBUTION (Cont'd)

HARRY DIAMOND LABORATORIES
ATTN 00100, COMMANDER/TECHNICAL DIR/TSO
ATTN CHIEF, 00210
ATTN CHIEF, DIV 10000
ATTN CHIEF, DIV 20000
ATTN CHIEF, DIV 30000
ATTN CHIEF, DIV 40000
ATTN CHIEF, LAB 11000
ATTN CHIEF, LAB 13000
ATTN CHIEF, LAB 15000
ATTN CHIEF, LAB 22000
ATTN CHIEF, LAB 21000
ATTN CHIEF, LAB 34000
ATTN CHIEF, LAB 36000
ATTN CHIEF, LAB 47000
ATTN CHIEF, LAB 48000
ATTN RECORD COPY, 94100
ATTN HDL LIBRARY, 41000 (5 COPIES)
ATTN HDL LIBRARY, 41000 (WOODBRIDGE)
ATTN CHAIRMAN, EDITORIAL COMMITTEE
ATTN TECHNICAL REPORTS BRANCH, 41300
ATTN LEGAL OFFICE, 97000
ATTN LANHAM, C., 00210
ATTN WILLIS, B., 47400
ATTN TORRIERI, D. J., (25 COPIES)